

$$\begin{aligned}p_1 &= (1 - f)x + fc \\p_2 &= (1 - f)c + fd \\p_3 &= (1 - f)d + fy \\p_{12} &= (1 - f)p_1 + fp_2 \\p_{23} &= (1 - f)p_2 + fp_3 \\p_{123} &= (1 - f)p_{12} + fp_{23}\end{aligned}$$