Leader Election in d-dimensional CA in time $o(diam^{1+\varepsilon})$

Michael Stratmann Thomas Worsch
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Leader Election in d-dimensional CA in time $O(diam \cdot (\log diam)^{d+1})$

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Overview

- Problem
- History
- Basic Idea
- Algorithm: selected aspects
  - overview
  - counters
  - graph
  - time complexity

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Problem

Given: d-dimensional grid, a subset of states {yellow, black, red}

Problem instance: initial configuration with

- a finite subset of yellow cells,
  connected with respect to von Neumann neighborhood
- surrounded by black cells
  not taking part in the computation

Task: Find a set of states and a local rule for a CA such that always exactly one cell is designated as *leader* or *queen bee*.

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History

- Smith (1971): *Queen Bee Problem*,
  2-dim., $\mathrm{O}\left(peri\right) \subseteq \mathrm{O}\left(vol\right) \subseteq \mathrm{O}\left(diam^2\right)$ (correct?)
- Vollmar (1979): simplified problem, 2-dim., $\mathrm{O}\left(peri\right)$
- Hemmerling (1979): d-dim., $\mathrm{O}\left(vol^5\right)$
- Beckers (1995): 2-dim., $\mathrm{O}\left(peri\right)$
- Mazoyer/Rémila (1998): 2-dim., $\mathrm{O}\left(peri\right)$
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Basic Idea

- Select some candidate cells in the first step.
- Use a total order on the candidates.
- Compare pairs of candidates, eliminating the "weaker" one.
- Elect the only remaining candidate as leader.

Most algorithms: lexicographic order on the coordinates,

$$\text{i.e.} \quad (x,y,z) \sqsubset (x',y',z') \quad \text{iff}$$
$$x < x' \quad \text{or} \quad x = x' \wedge y < y' \quad \text{or} \quad x = x' \wedge y = y' \wedge z < z'$$

Candidates: cells which are the strongest locally (von Neumann)



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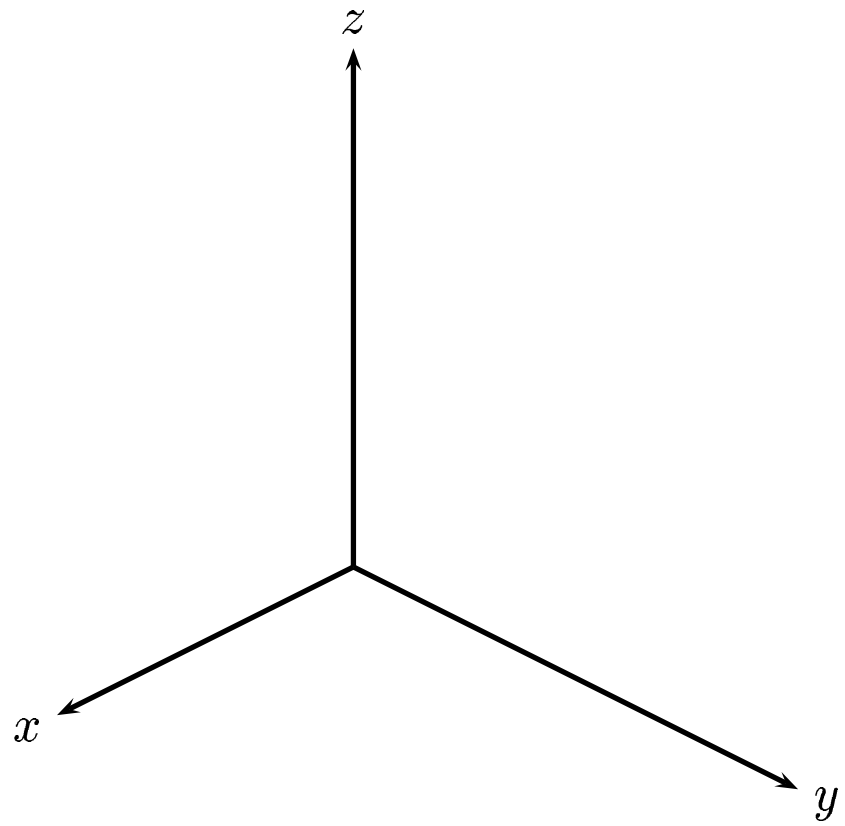

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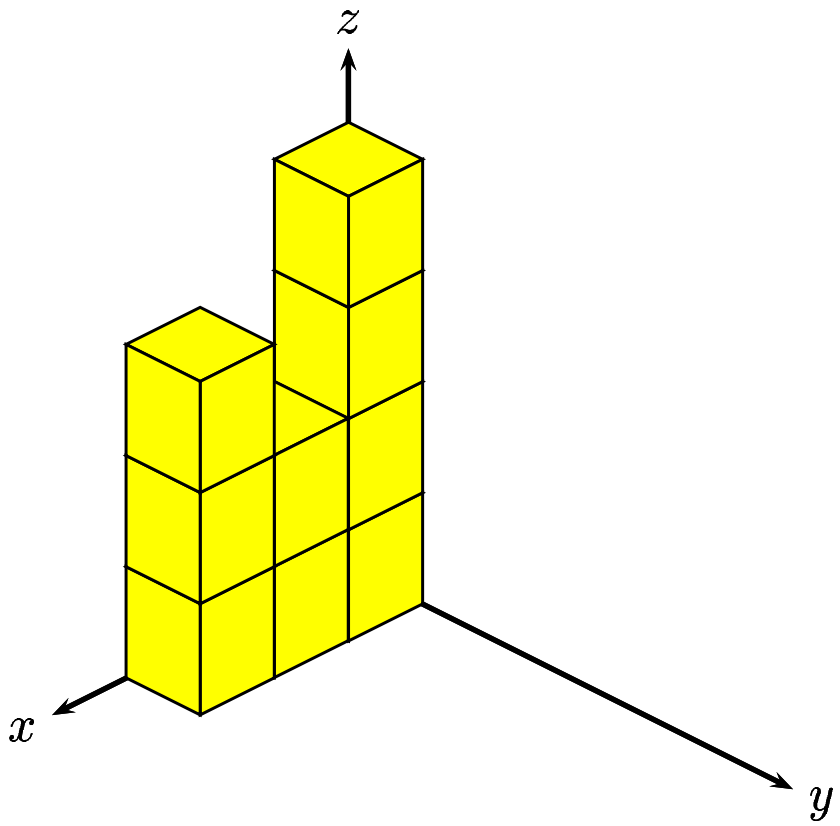

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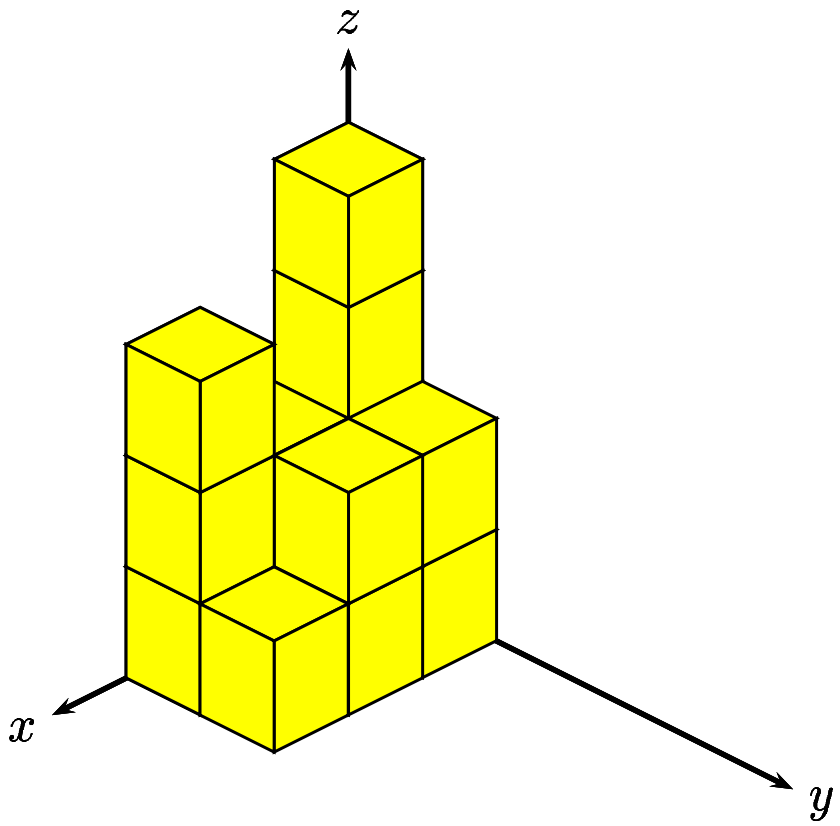

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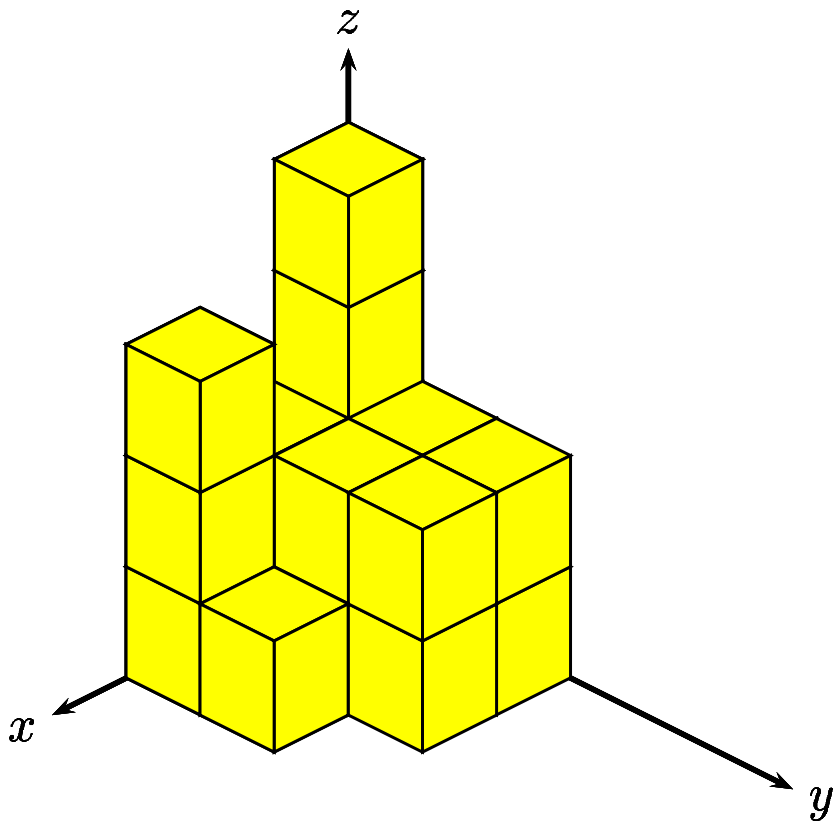

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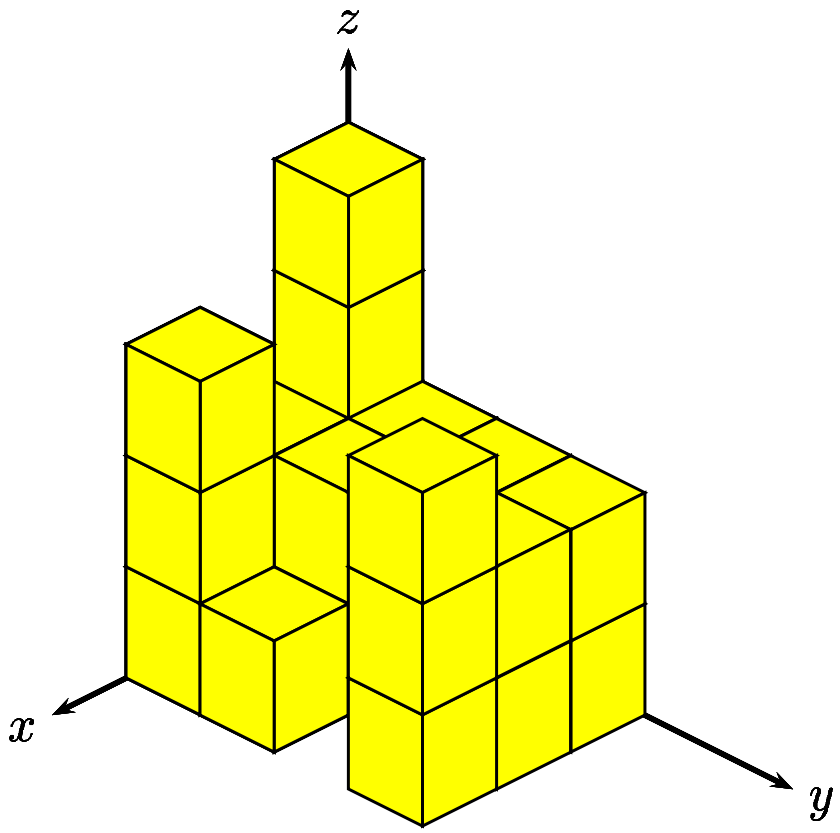

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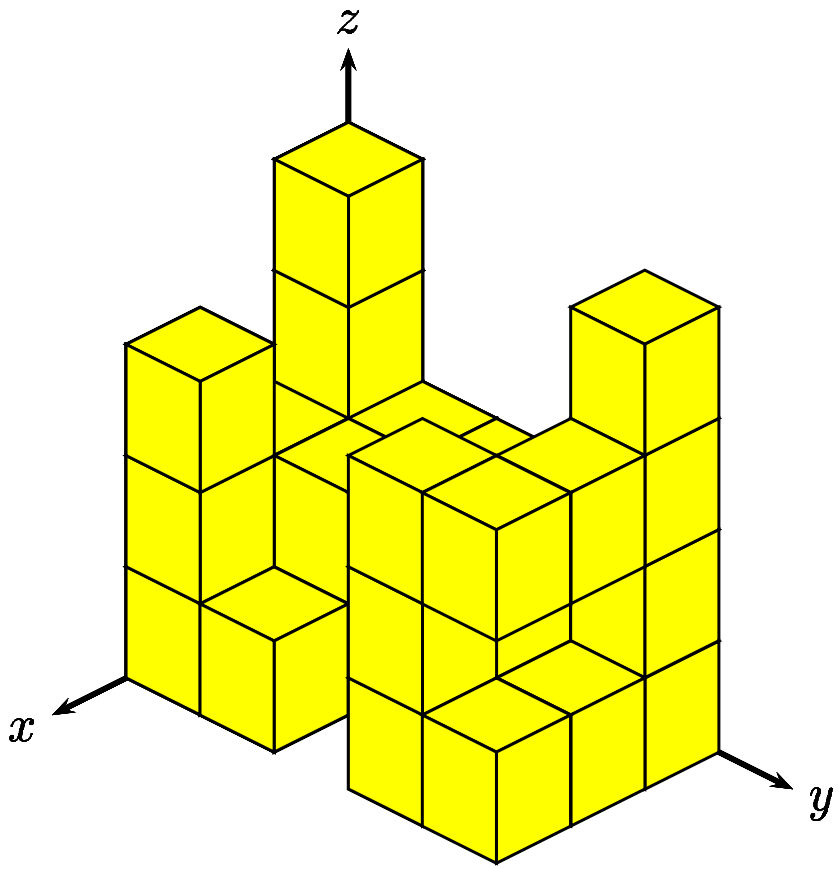

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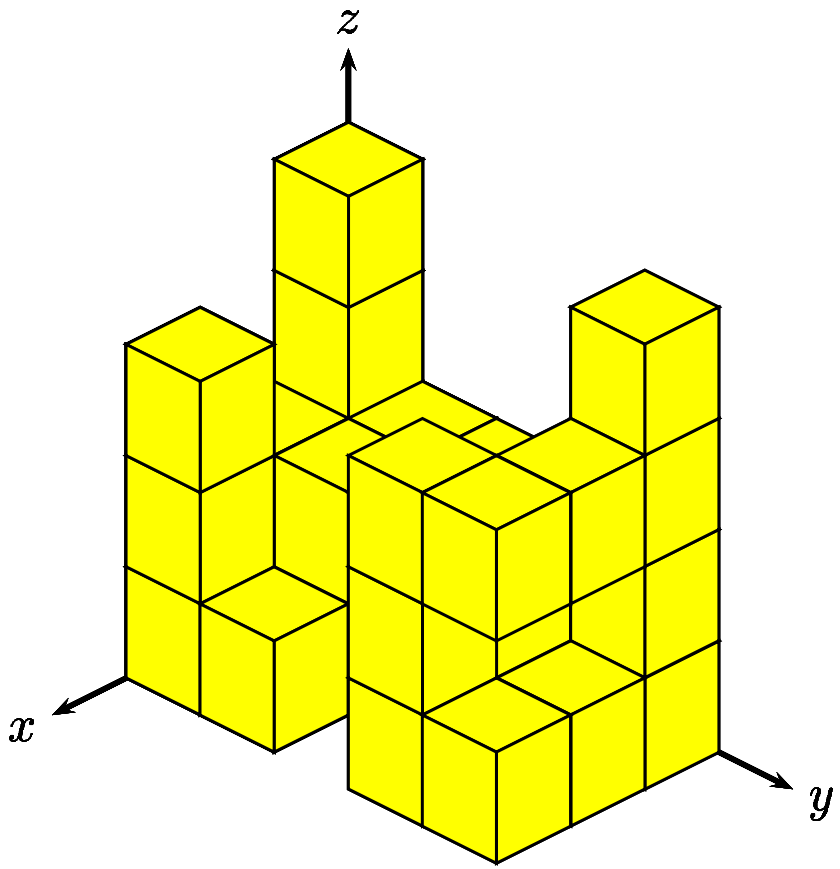

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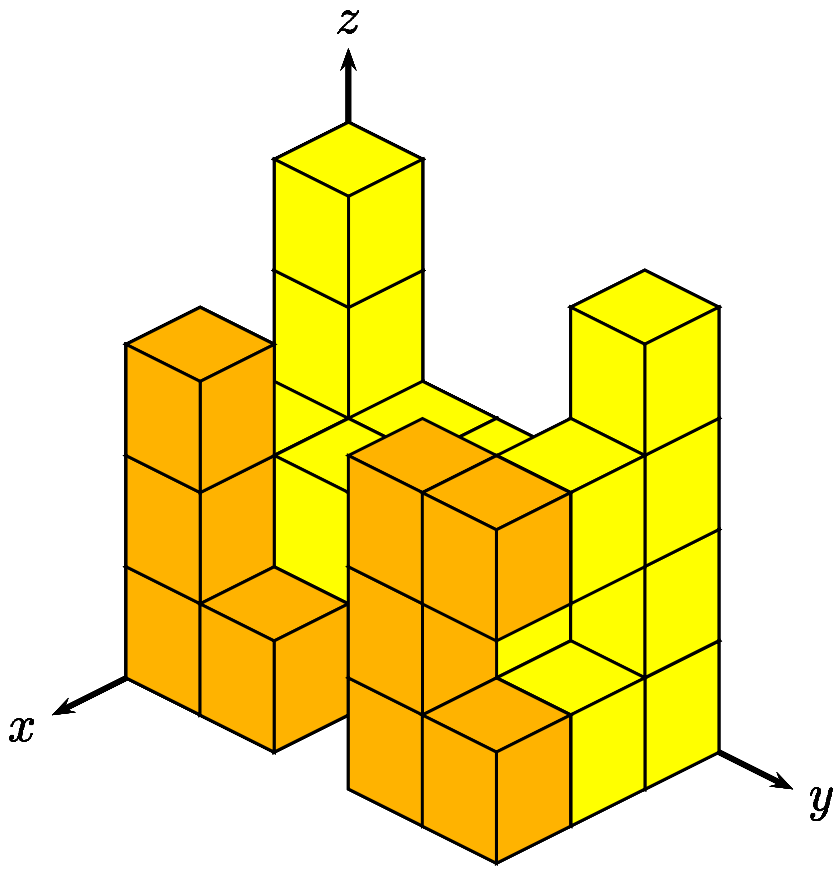

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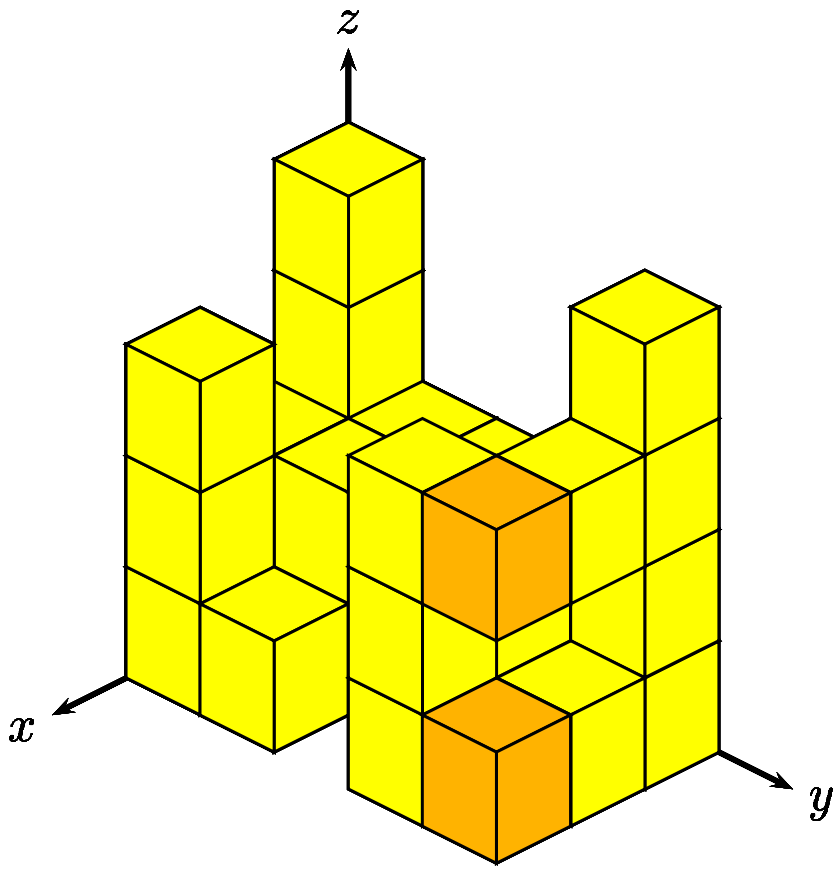

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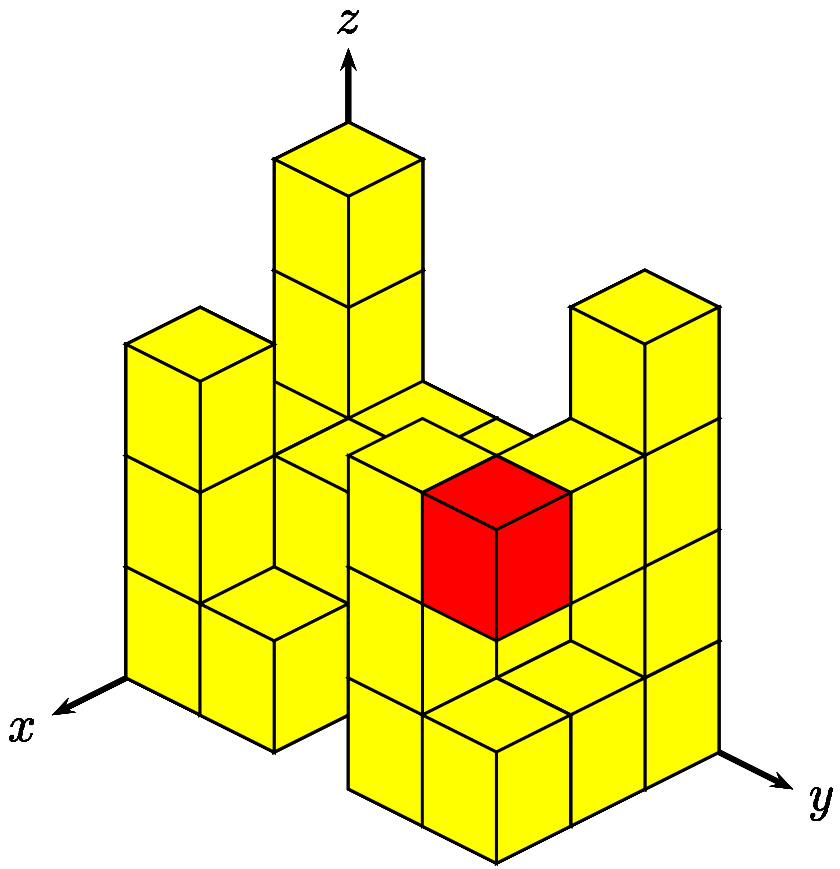

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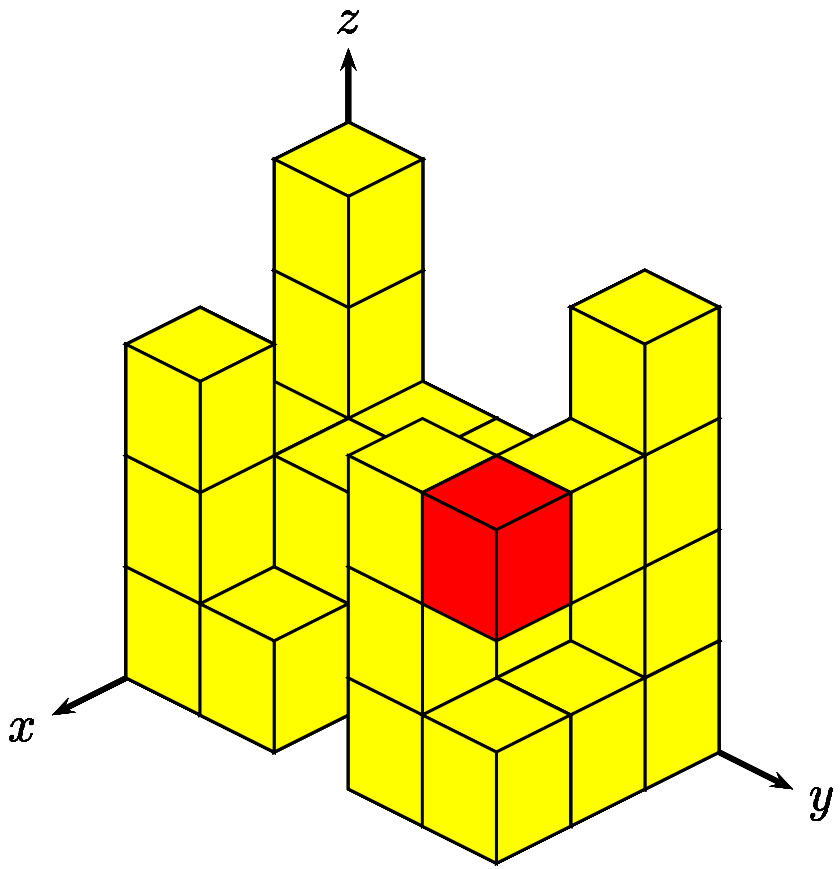

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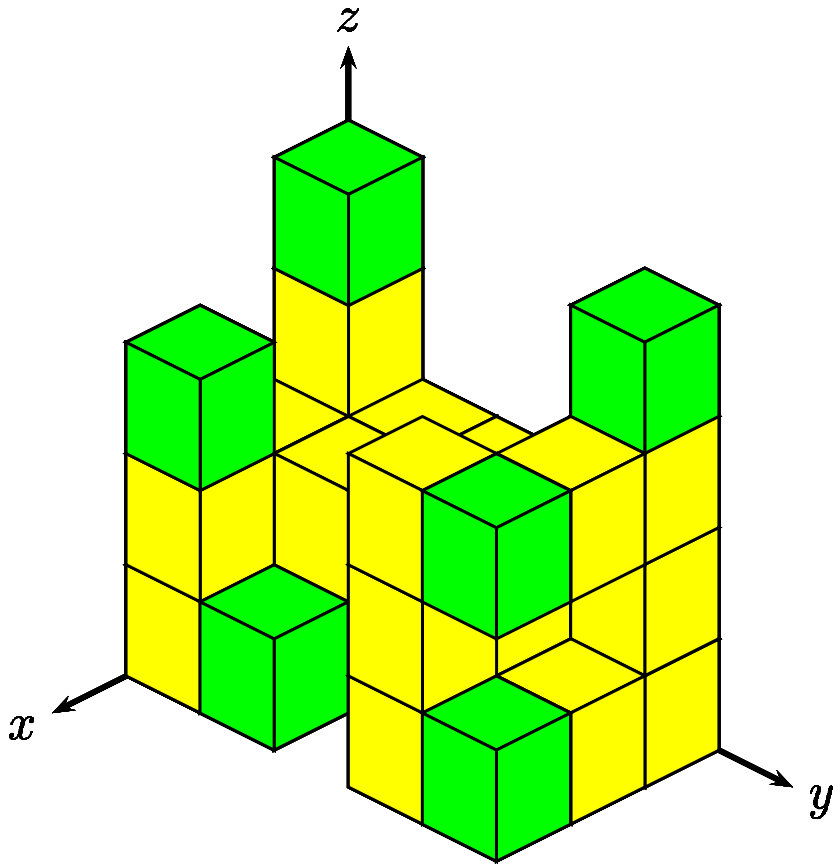

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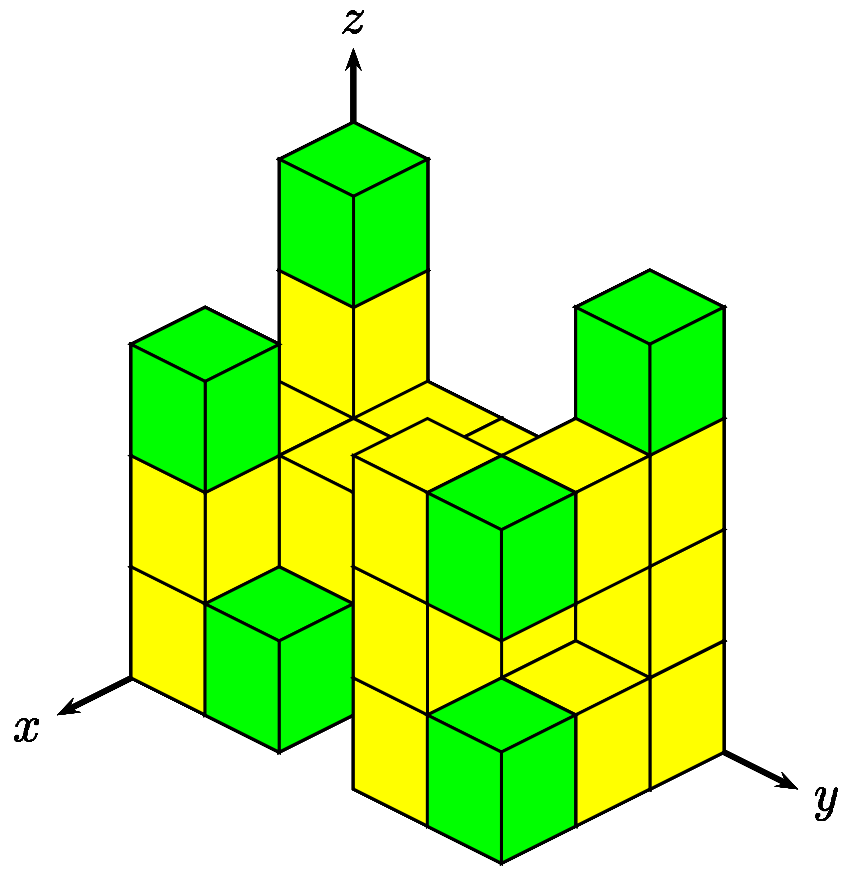
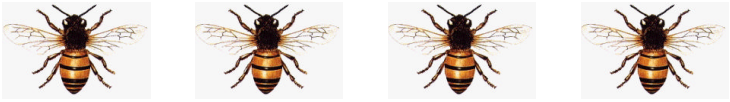


Problem: comparison of candidates

The Algorithm: selected aspects

- overview
- space counter
  - simple moving counter
  - generalization
- graph
  - initialization
  - deletion of edges
  - deletion of vertices
- time complexity



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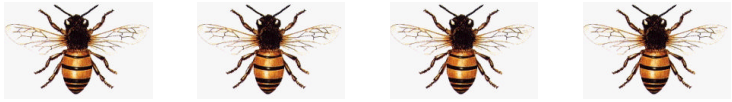

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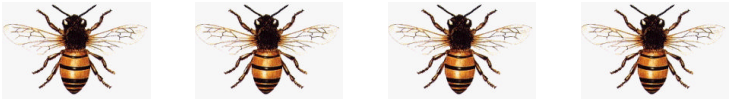

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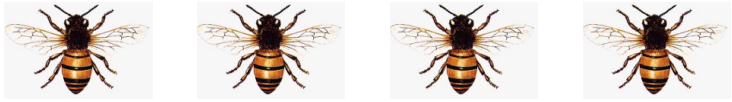

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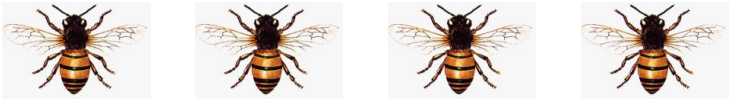

Algorithm: overview

- work on a graph, initially corresponding to the pattern
- shrink the graph until only one vertex is left: the leader

- build spanning trees starting at each candidate
- when two trees hit each other, they duel
  - the one originating at the stronger candidate wins
  - and spreads further into the territory of the weaker candidate
- when a tree hits the border, the branch dies

- each branch is constructed as the trail of a moving counter;
  it contains the relative distance to the candidate it started at

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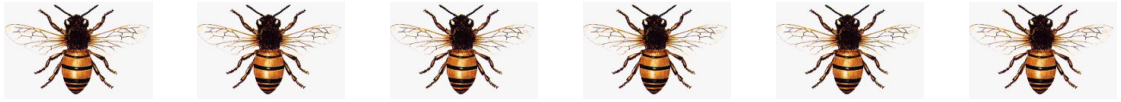
Counters: the simple case

	(1)												
	0												



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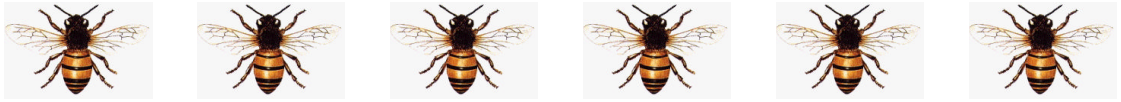

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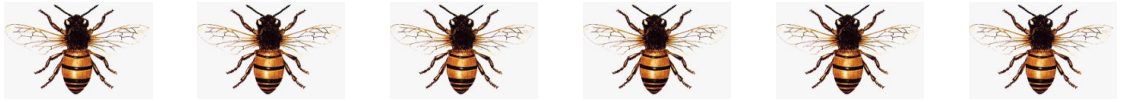

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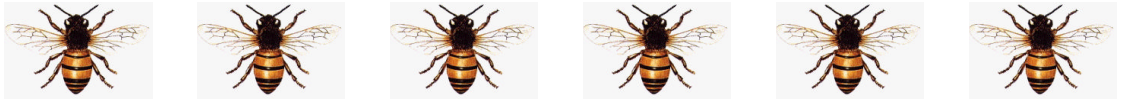

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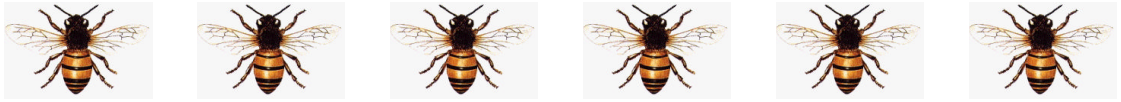

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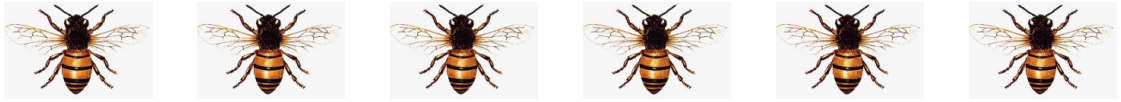

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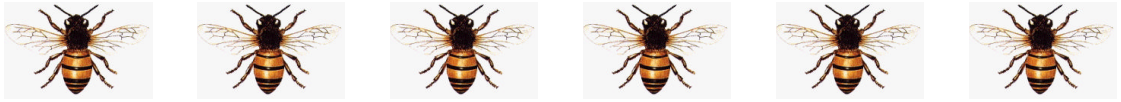

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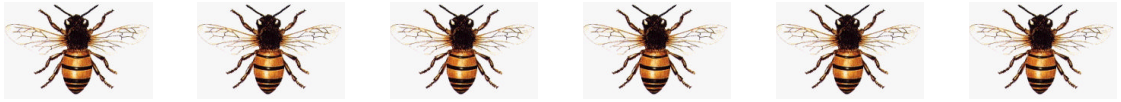

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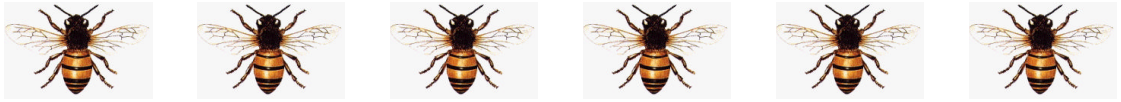

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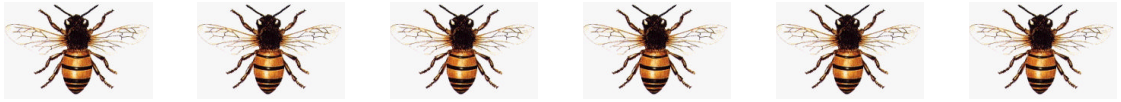
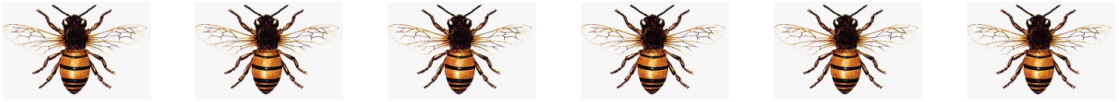


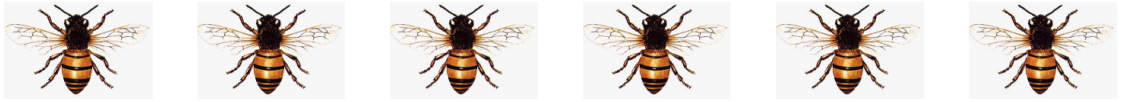
Counters: the simple case

								(1	0	1	1)		
								0	0	0	0		



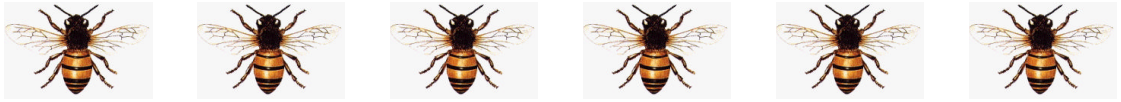
Counters: the simple case

									(1	0	1	0)	
									0	0	0	1	



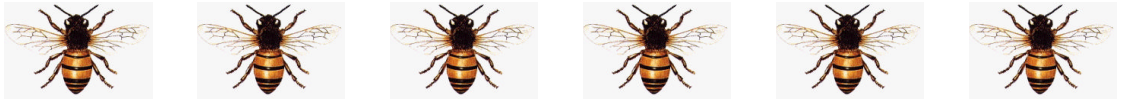
Counters: the simple case

										(1	0	0	
										0	0	1	



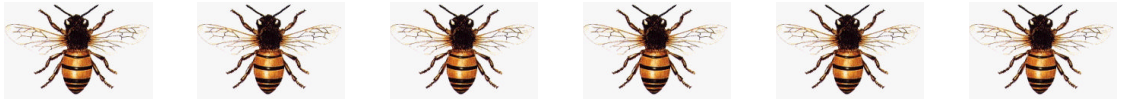
Counters: the simple case

											(1	1	
											0	0	



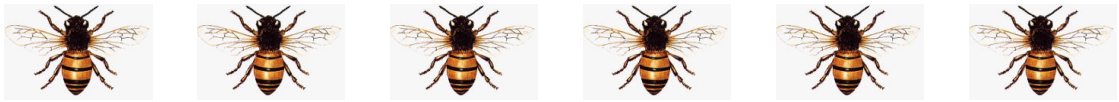
Counters: the simple case

												(1	
												0	



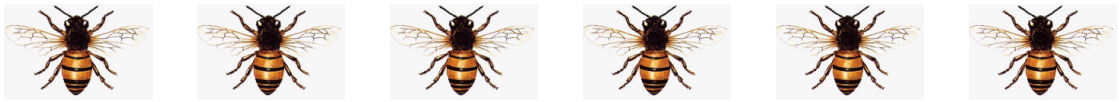
Counters: the simple case

1	2	3	4	5	6	7	8	9	10
(1)									
0									
	0)								
	(1								
	(1	1)							
	0	0							
		(1	0)						
		0	1						
			0	1)					
			(1	0					
			(1	0	0)				
			0	0	1				
				(1	1	1)			
				0	0	0			
					(1	1	0)		
					0	0	1		
						(1	0	1)	
						0	1	0	
							0	0	0)
							(1	0	1



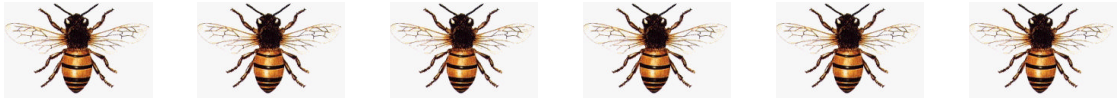
Counters: the simple case

1	2	3	4	5	6	7	8	9	10
(1)									
0									
	0)								
	(1								
	(1	1)							
	0	0							
		(1	0)						
		0	1						
			0	1)					
			(1	0					
			(1	0	0)				
			0	0	1				
				(1	1	1)			
				0	0	0			
					(1	1	0)		
					0	0	1		
						(1	0	1)	
						0	1	0	
							0	0	0)
							(1	0	1



Counters: the simple case

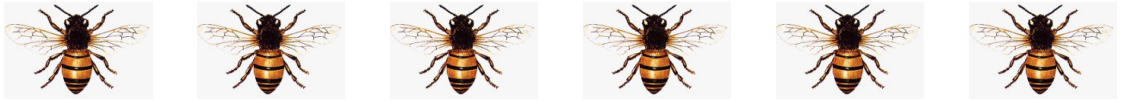
1	2	3	4	5	6	7	8	9	10
(1)									
0									
	0)								
	(1								
	(1	1)							
	0	0							
		(1	0)						
		0	1						
			0	1)					
			(1	0					
			(1	0	0)				
			0	0	1				
				(1	1	1)			
				0	0	0			
					(1	1	0)		
					0	0	1		
						(1	0	1)	
						0	1	0	
							0	0	0)
							(1	0	1



Counters: the simple case

	(1)												
	0												

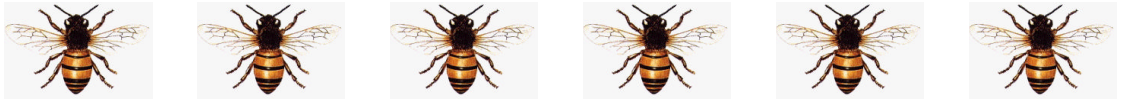
1



Counters: the simple case

		0)											
		(1											

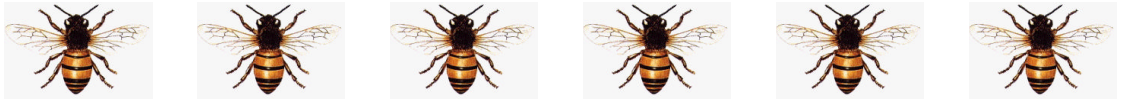
1 0



Counters: the simple case

		(1	1)										
		0	0										

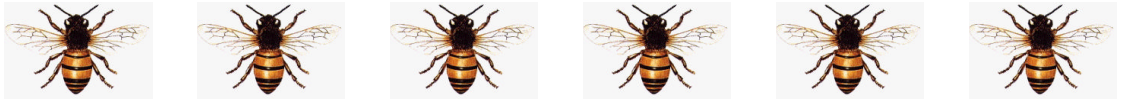
$$\begin{array}{ccc} 1 & 0 & 1 \\ & 1 & \end{array}$$



Counters: the simple case

			(1	0)									
			0	1									

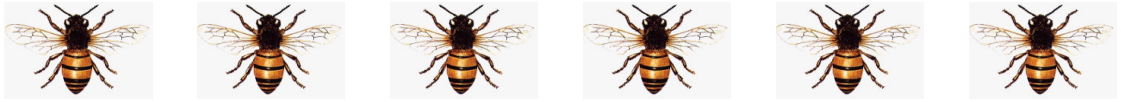
$$\begin{array}{cccc} 1 & 0 & 1 & 0 \\ & 1 & 1 & \end{array}$$



Counters: the simple case

				0	1)								
				(1	0								

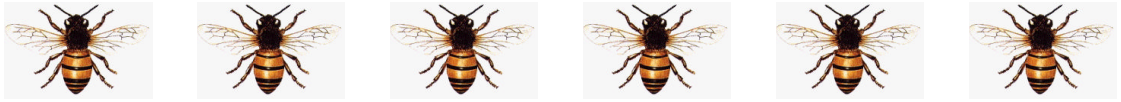
$$\begin{array}{ccccc} 1 & 0 & 1 & 0 & 1 \\ & 1 & 1 & 0 & \end{array}$$



Counters: the simple case

				(1	0	0)							
				0	0	1							

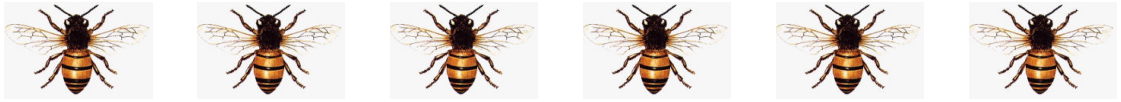
$$
\begin{array}{cccccc}
1 & 0 & 1 & 0 & 1 & 0 \\
 & 1 & 1 & 0 & 0 & \\
 & & & 1 & & \\
\end{array}
$$



Counters: the simple case

					(1	1	1)						
					0	0	0						

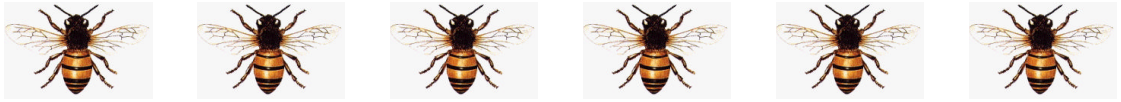
$$
\begin{array}{ccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 \\
 & 1 & 1 & 0 & 0 & 1 & \\
 & & & 1 & 1 & &
\end{array}
$$



Counters: the simple case

						(1	1	0)					
						0	0	1					

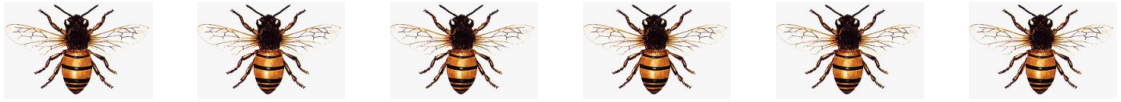
$$
\begin{array}{cccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
 & 1 & 1 & 0 & 0 & 1 & 1 & \\
 & & & 1 & 1 & 1 & & \\
\end{array}
$$



Counters: the simple case

							(1	0	1)				
							0	1	0				

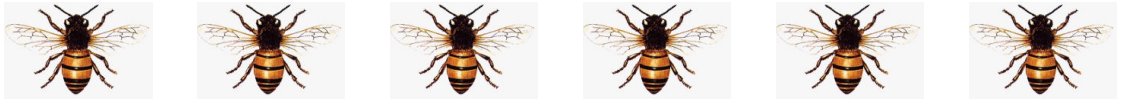
$$
\begin{array}{ccccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & \\
 & & & 1 & 1 & 1 & 1 & &
\end{array}
$$



Counters: the simple case

								0	0	0)			
								(1	0	1			

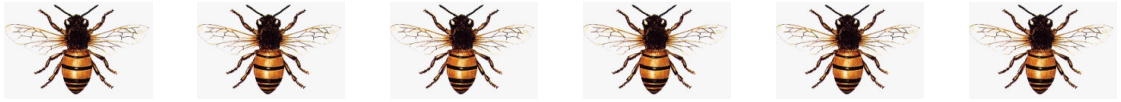
$$
\begin{array}{cccccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & \\
 & & & 1 & 1 & 1 & 1 & 0 & & \\
\end{array}
$$



Counters: the simple case

								(1	0	1	1)		
								0	0	0	0		

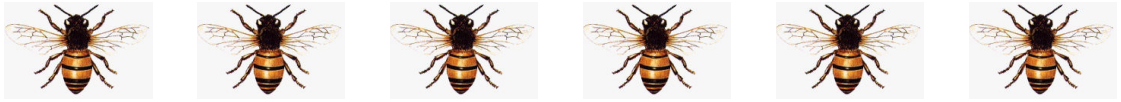
$$
\begin{array}{ccccccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & \\
 & & & 1 & 1 & 1 & 1 & 0 & 0 & & \\
 & & & & & & & 1 & & &
\end{array}
$$



Counters: the simple case

									(1	0	1	0)	
									0	0	0	1	

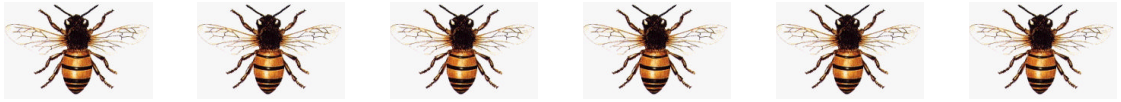
$$
\begin{array}{cccccccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & \\
 & & & 1 & 1 & 1 & 1 & 0 & 0 & 0 & & \\
 & & & & & & & 1 & 1 & & &
\end{array}
$$



Counters: the simple case

										(1	0	0	
										0	0	1	

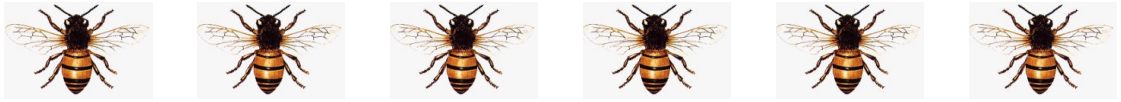
$$
\begin{array}{cccccccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
 & & & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & \\
 & & & & & & & 1 & 1 & 1 & & \\
\end{array}
$$



Counters: the simple case

											(1	1	
											0	0	

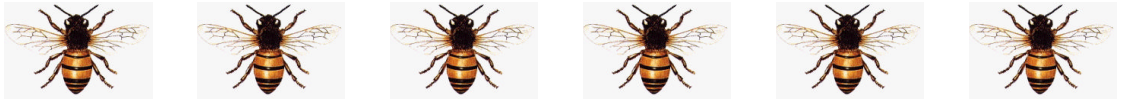
$$
\begin{array}{cccccccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
 & & & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\
 & & & & & & & 1 & 1 & 1 & 1 &
\end{array}
$$



Counters: the simple case

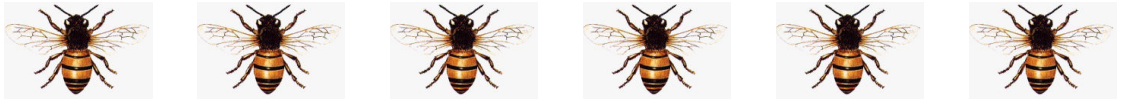
											(1	
											0	

$$
\begin{array}{cccccccccccc}
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
 & & & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\
 & & & & & & & 1 & 1 & 1 & 1 & 1
\end{array}
$$



Counters: as used in the algorithm

- counters for *integers*
  increment/decrement when moving in $+/-$ direction
- counters can be stretched and compressed by a factor of 2
- *space counter*: d such counters, one for each dimension
- space counters replicate at nodes with degree > 2: $\Longrightarrow$ bundles
- after moving to the next cell each counter waits for a time proportional to its length
- When two space counters meet, they duel:
  - copies of the contents are compared "bit by bit"
  - the winner moves on, the loser is destroyed



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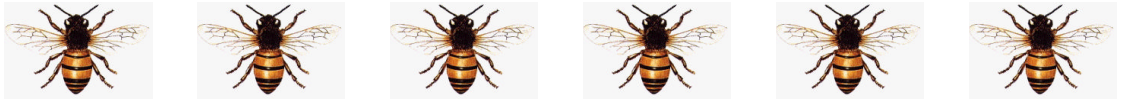

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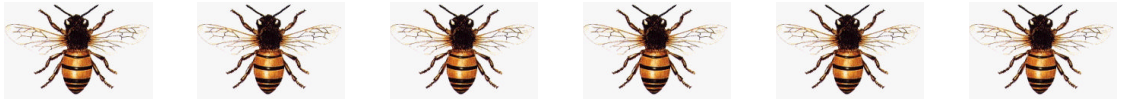

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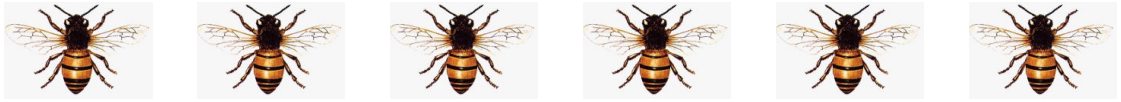

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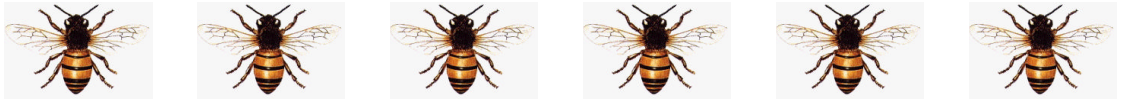

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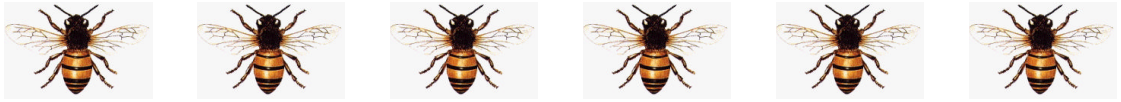

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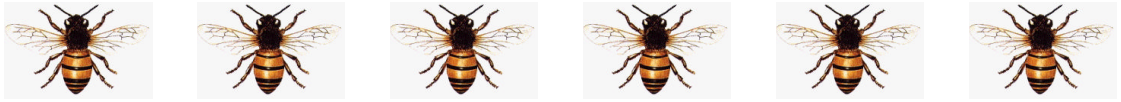

Graph

- each vertex correponds to a cell
- each vertex is stored in that cell
- an edge connects two vertices only if cells are immediate neighbors
- an edge is stored in the "cells" it connects

Initialization:

- a vertex for each cell
- an edge for each pair of neighboring cells

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Graph deletions

- a node and its incident edge are deleted
  - if they are the end of a branch which will not extend any more

- an edge is deleted,
  - if a loop is detected (duel of counters with identical contents)

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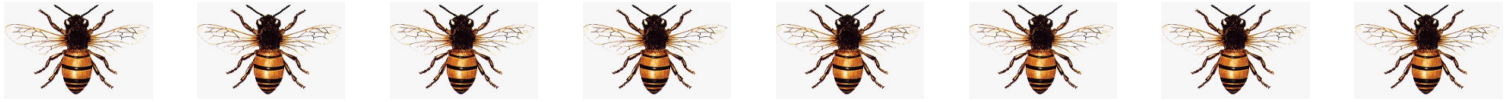
Nasty implementation details

much less than in Stratmann's original algorithm
(due to the more synchronized mode of operation of the counters)

Length of counters

The graph does not get disconnected.



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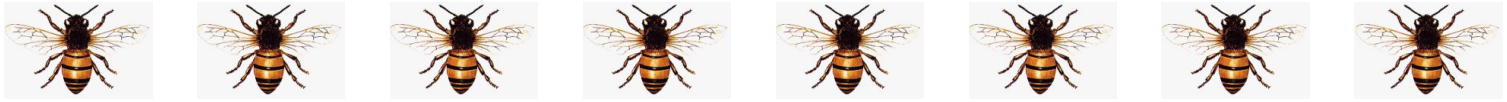

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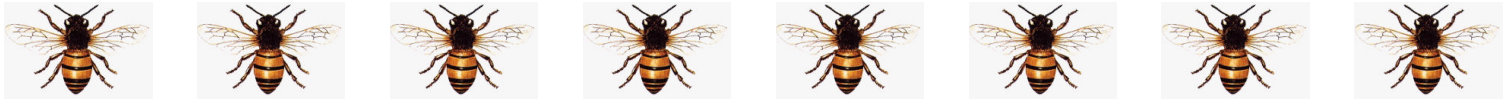
The graph does not get disconnected.



Time complexity

Claim 1:
The length of branches of the spanning trees is $\mathrm{O}(diam)$.

Claim 2:
It takes time $\mathrm{O}(\log diam)$ until a branch is extended by one edge.

Claim 3:
The product is the dominating summand of the running time.

Leader election in d-dimensional CA is possible in time

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The end